CPE 310C: Numerical Analysis for Engineers

Final Exam, May 23, 2017

- This is a **120-minute** CLOSED BOOK exam, with a total of **50 marks**. There are **33 questions** each one worth **1.5** marks, and **10 pages** (including this cover page).
- All your answers to multiple choice questions must be marked on this answer sheet. We will **not** take into consideration anything written on the question booklet or if multiple markings are made on the answer sheet. Make sure to mark only one answer.

GOOD LUCK

Total Grade
/50

Serial No

Student Name | Instructor & Class

cpe310-final-exam-spring2017 (9356)

1	A B C D	18	A B C D	27	A B C D
2	A B C D	19	A B C D	28	A B C D
3	A B C D	20	A B C D	29	A B C D
4	A B C D	21	A B C D	30	A B C D
5	A B C D	22	A B C D	31	A B C D
6	A B C D	23	A B C D	32	A B C D
7	A B C D	24	A B C D	33	A B C D
8	A B C D	25	A B C D		
9	A B C D	26	A B C D		

10	A B C D
11	A B C D
12	A B C D
13	A B C D
14	A B C D
15	A B C D
16	A B C D
17	A B C D

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Student ID

0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	7	7	7
8	8	8	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9	9	9

1. Using **Newton-Cotes integration** formula, estimate the value of $\int_0^{1.5} f(x)dx$ using the points $x_0 = 0, x_1 = 0.5, x_2 = 1, x_3 = 1.5$, given that $f(x_0) + f(x_3) = 6.75$ and $f(x_1) + f(x_2) = 2.25$.
   (A) 2.5313
   (B) 9.3323
   (C) 0.0000
   (D) 4.5000
2. Which of the following is always **TRUE** if $f(x)$ is a **quadratic polynomial**?
   (A) $\int_{-5}^{5} f(x)dx = f(-5) + 4f(0) + f(5)$
   (B) $\int_{-1}^{1} f(x)dx = f(-1) + 4f(0) + f(1)$
   (C) $\int_{-3}^{3} f(x)dx = f(-3) + 4f(0) + f(3)$
   (D) $\int_{-5}^{5} f(x)dx = 2f(-5) + 3f(-1) + 4f(0) + f(5)$
3. The **Heun's Method** (2^{nd} order Runge-Kutta) that is used to estimate y_{n+1} is given by:
   (A) $y_{n+1} = y_n + hk_2$
   (B) $y_{n+1} = y_n + h(\frac{1}{3}k_1 + \frac{2}{3}k_2)$
   (C) $y_{n+1} = y_n + h(\frac{2}{3}k_1 + \frac{1}{3}k_2)$
   (D) $y_{n+1} = y_n + h(\frac{1}{2}k_1 + \frac{1}{2}k_2)$
4. The following formula: $f'_{i+1} = \frac{1}{h}\left[(f_{i+1} - f_i) + \frac{1}{2}(f_{i+2} - 2f_{i+1} + f_i)\right]$ is a:
   (A) forward-difference formula
   (B) backward-difference formula
   (C) central-difference formula
   (D) none of the above

Use the following table for answer questions 5–6.

t	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
$f(t)$	1.0000	0.9960	0.9695	0.9068	$f(0.8)$	0.7071	0.6054	0.5168	0.4430	0.3826	$f(2.0)$

5. Given that $\int_{0.6}^{1.0} f(t)dt = 0.324447$, use the **Newton-Cotes Integration** formula to find $\boldsymbol{f(0.8)}$.
   (A) 0.7948
   (B) 0.8132
   (C) 0.8322
   (D) 0.8607

6. Given that $\sum_{i=1}^{10} \frac{f_i + f_{i+1}}{2} = 7.0072$, find $\int_0^2 f(t)dt$ using the **Composite Trapezoidal** rule.

(A) 1.1513
(B) 1.2473
(C) 1.3298
(D) 1.4014

7. The function $f(x)$ is continuous on its domain in terms of $\boldsymbol{k}$ certain function values which are given in the table below. If the **Composite Trapezoidal** rule approximation of $\int_5^{17} f(x)dx = 84$ with **three equal subintervals**, then what is the value of $\boldsymbol{k}$.

$\boldsymbol{x}$	5	9	13	17
$\boldsymbol{f(x)}$	6	$k+1$	$2k-1$	$2k$

(A) $\frac{9}{2}$
(B) $\frac{2}{9}$
(C) $\frac{1}{4}$
(D) $\frac{4}{1}$

8. Estimate the value of $\boldsymbol{f'(3.3)}$ with a **quadratic polynomial** (degree 2) that is constructed if we enter the table at $\boldsymbol{i = 2}$, given the difference table below:

$\boldsymbol{i}$	$\boldsymbol{x_i}$	$\boldsymbol{f_i}$	$\boldsymbol{\Delta f_i}$	$\boldsymbol{\Delta^2 f_i}$	$\boldsymbol{\Delta^3 f_i}$	$\boldsymbol{\Delta^4 f_i}$
0	1.30	3.669	3.017	2.479	2.041	1.672
1	1.90	6.686	5.496	4.520	3.713	3.058
2	2.50	12.182	10.016	8.233	6.771	5.562
3	3.10	22.198	18.249	15.004	12.333	
4	3.70	40.447	33.253	27.337		
5	4.30	73.700	60.590			
6	4.90	134.290				

(A) 27.875
(B) 27.113
(C) 25.342
(D) 28.128

9. Use **Simpson's $\frac{1}{3}$** method to evaluate the integral of $f(x)$ over the interval $[0.4, 0.6]$ using the data in the table below:

x	0.3	0.4	0.5	0.6
$f(x)$	1.23751	1.64250	1.35393	1.09533

(A) 0.415785

(B) 0.427844

(C) 0.271785

(D) 0.628749

10. Suppose that you are given the following data about a polynomial $f(x)$ of **unknown degree** where $f(0) = 2$, $f(1) = -1$, and $f(2) = 4$, the **coefficient of x^2** in $f(x)$, if all $\Delta^3 f_i$ are equal to **1** is:

(A) 3.5

(B) 2

(C) 1

(D) 0

11. The following data of the velocity of a body is given as a function of time:

Time, t (s)	4	7	10	13
Velocity, $v(t)$ (m/s)	22	24	37	46

The best estimate of the **distance** in meters covered by the body from $t = 4$ to $t = 13$ using **Simpson's $\frac{3}{8}$** rule would be:

Hint: the distance d is equal to $\int_{t_{\text{start}}}^{t_{\text{final}}} v(t)dt$.

(A) 200.000

(B) 282.375

(C) 102.334

(D) 500.000

12. Given $3\frac{dy}{dx} + 5\sqrt{y} = e^{0.1x}, y(0.3) = 5$ and using a step size of $h = 0.3$, the best estimate of $\frac{dy}{dx}(0.9)$ using the **Mid-Point Method** (2^{nd} order Runge-Kutta) most nearly is:

(A) -2.2473

(B) -2.2543

(C) -2.6188

(D) -3.2045

13. The value of $\int_0^{\pi} cos(\frac{\pi}{2} - x)dx$ using **Simpson's $\frac{3}{8}$** rule with **3** equally-spaced intervals.

(A) 2.00000

(B) 2.04052

(C) 2.09425

(D) 2.40200

14. A student finds the numerical value of $\frac{d}{dx}(e^x) = 18.20445$ at $x = 3$ using a step size of $h = 0.2$. Which of the following methods did the student use to conduct the differentiation?

(A) forward-difference formula

(B) backward-difference formula

(C) central-difference formula

(D) divided-difference method of degree 10

15. Suppose that **least-squares approximations** is used to fit some tabular data with a polynomial of degree **5**, we found the errors as shown in the table below. The value for the variance σ^2 is:

$\boldsymbol{i}$	1	2	3	4	5	6	7	8	9	10
$\boldsymbol{e_i}$	0.02	0.02	0.02	0.01	0.03	0.00	0.01	0.01	0.02	0.02

(A) 0.0032

(B) 0.16

(C) 0.04

(D) 0.0008

16. Estimate the value of $\boldsymbol{f''(4)}$ using a polynomial of degree **three** using the data in the table below and the **divided-difference** method.

$\boldsymbol{x}$	-2	0	2	3	6
$\boldsymbol{f'(x)}$	7	-1	7	17	71

(A) 16

(B) 0

(C) 31

(D) 52

17. The value of $\int_3^{19} f(x)dx$ by using **Simpson's $\frac{1}{3}$** rule with **2** equally-spaced intervals is estimated as 702.039. The estimate of the same integral using **4** equally-spaced intervals is most nearly:

(A) $702.039 + \frac{8}{3}[2f(7) - f(11) + 2f(15)]$

(B) $\frac{702.039}{2} + \frac{8}{3}[2f(7) - f(11) + 2f(15)]$

(C) $702.039 + \frac{8}{3}[2f(7) + 2f(15)]$

(D) $\frac{702.039}{2} + \frac{8}{3}[2f(7) + 2f(15)]$

18. Say we are given a differential equation $\frac{dy}{dx} = f(x, y)$ for some function $f(x, y)$ and we have the initial condition $y(x_0) = x_0$. Which one of the following is the **correct** formula to compute $y(x_2)$ using **Simple Euler** Method?

(A) $y_2 = x_0 + hf(x_0, y_0)$

(B) $y_2 = x_0 + hf(x_0, y_0) + hf(x_1, x_0 + hf(x_0, y_0))$

(C) $y_2 = x_1 + hf(x_1, y_1) + hf(x_0, x_1 + hf(x_1, y_1))$

(D) $y_2 = x_0 + hf(x_0, y_0)$

19. The velocity of a body is given by $v(t) = \begin{cases} 2t, & 1 \le t \le 5 \\ 5t^2 + 3 & 5 \le t \le 14 \end{cases}$ where t is given in seconds, and v is given in m/s. Use the proper combination of **Simpson's $\mathbf{\frac{1}{3}}$** rule and **Composite Trapezoidal** method with step size $h = 1$ to find the **distance** covered by the body from $t = 3$ to $t = 9$ seconds. **Hint:** the distance d is equal to $\int_{t_{\text{start}}}^{t_{\text{final}}} v(t)dt$.

    (A) $979.7m$

    (B) $1034.7m$

    (C) $1260.9m$

    (D) $5048.9m$

20. If $y' = 3x^5 - 2x^4 + 3x - 10$ and $y(0) = -10$, then how many terms of the **Taylor-Series** is exactly needed to evaluate $y(1)$?

    (A) 6

    (B) 7

    (C) 8

    (D) 9

21. Use the **Ralston's Method** (2^{nd} Order Runge-Kutta) to evaluate $y(0.2)$ for $y' = x + y^2$ using an interval of 0.2, given that $y(0) = 0$.

    (A) 0.15

    (B) 0.015

    (C) 0.01

    (D) 0.2

22. Suppose that a polynomial of degree **9** is used to fit an evenly-spaced tabular data with $h = 0.5$. What is the coefficient of $\boldsymbol{x^9}$, if you knew that $\Delta^9 P_0(x) = 2126.25$?

    (A) 2

    (B) 3

    (C) 4

    (D) 5

23. Suppose that we have applied the **Simple Euler** method $\boldsymbol{n}$ times to the following differential equation $y' = y$ with the initial condition $y(0) = 1$ and step size $h = 1$. After n iterations, $y(x_n) = 1048576$. Based on these information, the number of iterations n used was:

    (A) 6

    (B) 10

    (C) 20

    (D) 33

24. For $f(x) = x^4 cos(x)$, find the **absolute value of the error** $\left|f'_{\text{actual}}(5.0) - f'_{\text{approximate}}(5.0)\right|$ in calculating $f'(5.0)$ to 3 decimal places using the **central-difference formula** with 3 points and $h = 0.1$. In all your calculations, round to 3 decimal places.

(A) 0.172

(B) 0.214

(C) 0.009

(D) 0.016

25. Use the first **5** terms of the **Taylor-Series** to evaluate $y(0.4)$ for $y'' = xy$, given that $y(0) = 1$ and $y'(0) = 0$.

(A) 0.0000

(B) 1.0000

(C) 1.0107

(D) 1.1666

26. Given the function $f(x) = \frac{1}{2}\int_0^x e^{-t}dt$. The approximate value of $f(2.0)$ using the **first three terms** of the **Taylor-Series** around $x = 0$ is:

(A) -0.75225

(B) 0.00032

(C) 0.00000

(D) 1.00000

27. A scientist found that the data in the table below can be exatly fit ($\sum e_i^2 = 0$) with **least-square line** of the form $f(x) = a_0 + a_1 x$, then the value of $f(18)$ is equal to:

$\boldsymbol{x}$	0	5	10	11	12	18
$\boldsymbol{f(x)}$	-5	-2.5	0	0.5	1	$f(18)$

(A) 4

(B) -2.5

(C) 4.5

(D) 15

28. To solve $\frac{dy}{dx} = 2y - 2$, given $y(4.5) = 3$, $h = 0.25$, the **number of required steps** to find $y(50)$ using $\mathbf{4^{th}}$ **Order Runge-Kutta** method is:

(A) 146

(B) 150

(C) 182

(D) 200

29. For $y'' = x^2 + y$, find $\boldsymbol{y^{iv}(0)}$, given that $y = -1$ when $x = 0$.

(A) -2

(B) -1

(C) 0

(D) 1

30. Given $f(3) = 6, f'(3) = 8, f''(3) = 11$, and all other higher-order derivatives of $f(x)$ are zero at $x = 3$, and assuming the function and all its derivatives exist and are continuous between $x = 3$ and $x = 7$, use the **Taylor-Series** method to find the value of $f(7)$.

(A) 38.000

(B) 79.500

(C) 126.00

(D) 331.50

31. Given the difference table below, if $\sum_{i=0}^{2} \Delta^4 f_i = 0.283$, then the value of $\boldsymbol{\Delta^3 f_3}$ is:

$\boldsymbol{i}$	$\boldsymbol{x_i}$	$\boldsymbol{f_i}$	$\boldsymbol{\Delta f_i}$	$\boldsymbol{\Delta^2 f_i}$	$\boldsymbol{\Delta^3 f_i}$
0	0.0	0.000	0.203	0.017	0.024
1	0.2	0.203	0.220	0.041	0.044
2	0.4	0.423	Δf_2	0.085	0.096
3	0.6	0.684	0.346	$\Delta^2 f_3$	$\boldsymbol{\Delta^3 f_3}$
4	0.8	1.030	Δf_4	0.488	
5	1.0	$f(x_5)$	1.015		
6	1.2	2.572			

(A) 0.517

(B) 0.488

(C) 0.283

(D) 0.307

32. Let $y'' + y^2 = x$, which of the following statements is **correct**?

(A) This is a first-order differential equation. We need one initial condition to solve it.

(B) This is a first-order differential equation. We need two initial conditions to solve it.

(C) This is a second-order differential equation. We need one initial condition to solve it.

(D) This is a second-order differential equation. We need two initial conditions to solve it.

33. Use the **Modified-Euler Method** to evaluate $y(0.2)$ for $y' = x + y^2$ using an interval of 0.1, given that $y(0) = 0$.

(A) 0.0055000

(B) 0.0100025

(C) 0.0150025

(D) 0.0200125

Cheat Sheet

Divided Difference Interpolating Polynomial

$P_n(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) + \cdots + a_n(x - x_0)(x - x_1)\cdots(x - x_{n-1})$

Newton-Gregory Forward Interpolating Polynomial

$$P_n(x) = f_i + s\Delta f_i + \frac{s(s-1)}{2!}\Delta^2 f_i + \frac{s(s-1)(s-2)}{3!}\Delta^3 f_i + \cdots + \frac{s(s-1)\cdots(s-n+1)}{n!}\Delta^n f_i$$

Normal Equations using Least Line Squares Approxmiation of the Form: $y = ax + b$

$a\sum x_i^2 + b\sum x_i = \sum x_i Y_i$

$a\sum x_i + bN = \sum Y_i$

Derivatives from Divided Difference Table

$$P_n'(x) = f[x_0, x_1] + f[x_0, x_1, x_2]\sum_{i=0}^{1}\frac{(x - x_0)(x - x_1)}{(x - x_i)} + \cdots + f[x_0, \cdots, x_n]\sum_{i=0}^{n-1}\frac{(x - x_0)(x - x_1)\cdots(x - x_{n-1})}{(x - x_i)}$$

Derivatives from Difference Table: $P_n'(x) = \frac{1}{h}\left[\Delta f_i + \sum_{j=2}^{n}\left(\sum_{k=0}^{j-1}\prod_{\substack{l=0\\ l\neq k}}^{j-1}(s - l)\right)\frac{\Delta^j f_i}{j!}\right]$

Trapezoidal Rule: $\int_a^b f(x)dx = \frac{h}{2}\left[f_a + 2f_1 + 2f_2 + \cdots + 2f_{n-1} + f_b\right]$

Simpson's $\frac{1}{3}$ Rule: $\int_a^b f(x)dx = \frac{h}{3}\left[f_a + 4f_1 + 2f_2 + 4f_3 + \cdots + 4f_{n-1} + f_b\right]$

Simpson's $\frac{3}{8}$ Rule: $\int_a^b f(x)dx = \frac{3h}{8}\left[f_a + 3f_1 + 3f_2 + 2f_3 + \cdots + 2f_{n-3} + 3f_{n-2} + 3f_{n-1} + f_b\right]$

The Taylor-Series: $y(x) = y(x_0) + y'(x_0)(x - x_0) + y''(x_0)\frac{(x - x_0)^2}{2!} + y'''(x_0)\frac{(x - x_0)^3}{3!} + \cdots$

Simple Euler Method: $y_{n+1} = y_n + hy_n'$

Modified Euler Method: $y_{n+1} = y_n + h\frac{y_n' + y_{n+1}'}{2}$

2^{nd} **Order Runge-Kutta Method:** $y_{n+1} = y_n + h\left(ak_1 + bk_2\right)$

$k_1 = f(x_n, y_n), \quad k_2 = f(x_n + \alpha h, y_n + \beta k_1 h), \quad a + b = 1, \quad \alpha b = \frac{1}{2}, \quad \beta b = \frac{1}{2}$

4^{th} **Order Runge-Kutta Method:** $y_{n+1} = y_n + \frac{h}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)$

$k_1 = f(x_n, y_n), \quad k_2 = f(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_1 h), \quad k_3 = f(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_2 h), \quad k_4 = f(x_n + h, y_n + k_3 h)$

Course Exit Survey

Dear CPE 310A (Numerical Analysis for Engineers) Student,

Please indicate, in a scale from 1 to 5, how much you feel each of the following course learning outcomes has been achieved

1: Strongly not achieved   2: Not achieved   3: Neutral   4: Achieved   5: Strongly achieved

Course Learning Outcomes		1	2	3	4	5
CLO1	An ability to comrehend the difference between analytical and numerical solution of a mathematical problem					
CLO2	An ability to recognize different types of computational error					
CLO3	An ability to comrehend and apply the solution of nonlinear equations using basic numerical methods.					
CLO4	An ability to comrehend the notion of computational cost of an algorithm (expressed using rates of convergence)					
CLO5	An ability to comrehend and use the basics of linear algebra to solve sets of equations numerically					
CLO6	An ability to use and compare the common methods for solving sets of linear and nonlinear equations					
CLO7	An ability to comrehend the notion of interpolation and apply numerical method for curve fitting					
CLO8	An ability to comrehend and apply numerical methods for differentiation and integration.					
CLO9	An ability to comrehend and apply numerical methods for solving Ordinary Differential Equations					
CLO10	An ability to Use MATLAB programming language to apply all numerical methods learned in class.					