Yarmouk University

Hijjawi Faculty for Engineering Technology

Department of Computer Engineering

CpE310C: Numerical Analysis for Engineers

'First Exam'

<table>
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<tr><td colspan="6"></td><td>الرقم الجامعي</td></tr>
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Bisection Method	$x_2 = (x_0 + x_1)/2$
Secant Method/ False Position	$x_2 = x_0 - f(x_0) * \frac{(x_0 - x_1)}{f(x_0) - f(x_1)}$
Newton Method	$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$
Fixed-Point Iteration	$x_{n+1} = g(x_n)$

Question	**1**	**2**	**3**	**4**	**Total**
Grade					

Question 1: (11 points)

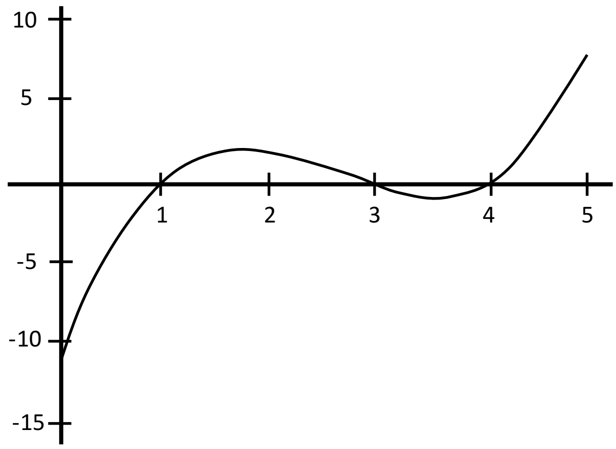
Question	1	2	3	4	5	6	7	8	9	10	11
Answer	A	D	B	E	D	A	D	G	B	G	C

1. Which of the following statements is CORRECT?
   A. Accuracy refers to how closely a computed or measured value agrees with the true value.
   B. Computers can compute the root for $f(x) = x^2 - 2 = 0$ with an actual error equals to zero.
   C. Our calculations for computing the area of a circle is always exact using the relation πr^2 where r is the radius of a circle.
   D. A and B
   E. A and C
   F. A, B, and C
   G. None of the above.

2. Which of the following methods of finding roots of nonlinear equations falls under the category of bracketing methods

   A. Bisection and Newton.
   B. Secant and Newton
   C. Secant and False position.
   D. False position and Bisection.
   E. Fixed-point iteration and Newton.
   F. Secant and Fixed-point iteration.
   G. None of the above.

3. The Bisection method is applied to the function $f(x) = (x-1)\,(x-3)\,(x-4)$ (graph given below). The initial guess interval used is $[0, 5]$. Which root will the method converge to:



   A. 0
   B. 1
   C. 2
   D. 3
   E. 4
   F. 5
   G. Cannot apply Bisection method because we will have $f(x_1)f(x_2) > 0$ at $[1.5, 2]$.

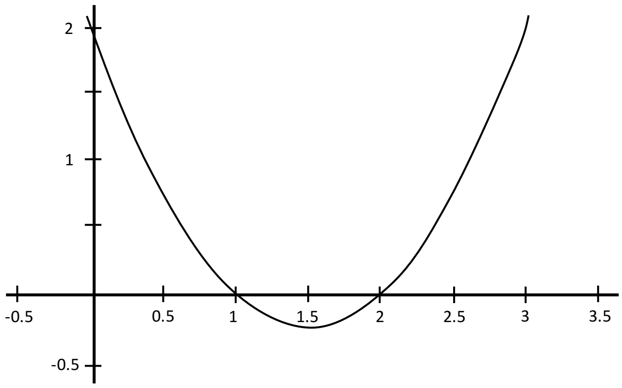
4. When finding the root of $\sin(x+\frac{\pi}{2})=0$, [x **in radians**] using the Secant method, the following choice of initial guesses would not be appropriate:
   A. $\left[\frac{\pi}{4},\ \frac{\pi}{3}\right]$
   B. $\left[\frac{\pi}{4},\ \frac{3\pi}{4}\right]$
   C. $\left[\frac{\pi}{3},\ \frac{2\pi}{3}\right]$
   D. $[1.3,\ 1.5]$
   E. $\left[-\frac{\pi}{8},\ \frac{\pi}{8}\right]$
   F. $[(-\frac{\pi}{2}-0.2),\ (-\frac{\pi}{2}+0.2)]$
   G. Secant method won't work because there are multiple roots.

5. The newly predicted root for false-position and secant method is given by:
$$x_{i+1}=x_i-f(x_i)\frac{x_i-x_{i-1}}{f(x_i)-f(x_{i-1})}$$
the major **difference** between the two methods is:
   A. False-position method is not guaranteed to converge.
   B. Secant method is guaranteed to converge
   C. Secant method requires the 2 initial guesses x_{i-1} *and* x_i to satisfy $f(x_{i-1})*f(x_i)<0$
   D. False-position method requires the 2 initial guesses x_{i-1} *and* x_i to satisfy $f(x_{i-1})*f(x_i)<0$
   E. A and D
   F. B and C
   G. No difference between the two method as both use the same predicted root formula: $x_{i+1}=x_i-f(x_i)\frac{x_i-x_{i-1}}{f(x_i)-f(x_{i-1})}$

6. Apply Newton's Method to find the roots of $f(x)=e^x$ starting from $x_0=5$. The condition $|f(x)|<\varepsilon$, where ε is a tolerance, is used as the stopping criterion. How many iterations are required to reach a tolerance of 10^{-18}?
   A. 47
   B. 41
   C. 40.00764
   D. 180
   E. 174
   F. Cannot be determined as we are missing the value for x_1.
   G. Newton's method cannot be applied because $f'(x)$ does not exist at $x=0$.

7. We want to solve the cubic equation $f(x)=x^3-3x^2+7x-5$ using the Fixed-Point Iteration Method. If we use the arrangement $x=\frac{-1}{7}x^3+\frac{3}{7}x^2+\frac{5}{7}$, which of the following starting points will lead to an oscillatory convergence?
   A. $x_0=4$
   B. $x_0=-1$
   C. $x_0=0.5$
   D. $x_0=-0.5$
   E. $x_0=-3$
   F. $x_0=1$
   G. The arrangement $g(x)=\frac{-1}{7}x^3+\frac{3}{7}x^2+\frac{5}{7}$ will always diverge for all x

8. Which of the following methods can be used to find root for $f(x) = x^2 - 3x + 2$ which is plotted below:



   A. Bisection method with initial guess interval of $[0, 3]$
   B. Secant method with initial guess interval of $[0, 3]$
   C. False-position method with initial guess interval of $[0, 3]$
   D. Newton's method with $x_0 = 1.5$
   E. Fixed point iteration, where $g(x) = \frac{x^2+2}{3}$ and $x_0 = 3$.
   F. Cannot determine as $f(x)$ has complex roots.
   G. None of the above

9. Which of the following is a unit vector?
   A. $[\frac{1}{2} \quad \frac{1}{2}]^T$
   B. $[(2+2i) \quad (2-2i) \quad 1]^T$
   C. $[1 \quad 1 \quad 1 \quad 1]^T$
   D. $[(4-3i) \quad (4+3i) \quad (3-4i) \quad (3-4i)]^T$
   E. $\begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix}$
   F. $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$
   G. None of the given choices

10. Which of the following statements is not correct?
   A. A*I = A
   B. $(A * B)^T = B^T * A^T$
   C. $\det(I_4) = 1$
   D. A+A=2A
   E. $AB \neq BA$, if A is a lower triangular matrix and B is an upper triangular matrix.
   F. $(I_4)^T = I_4$
   G. I – A = –A

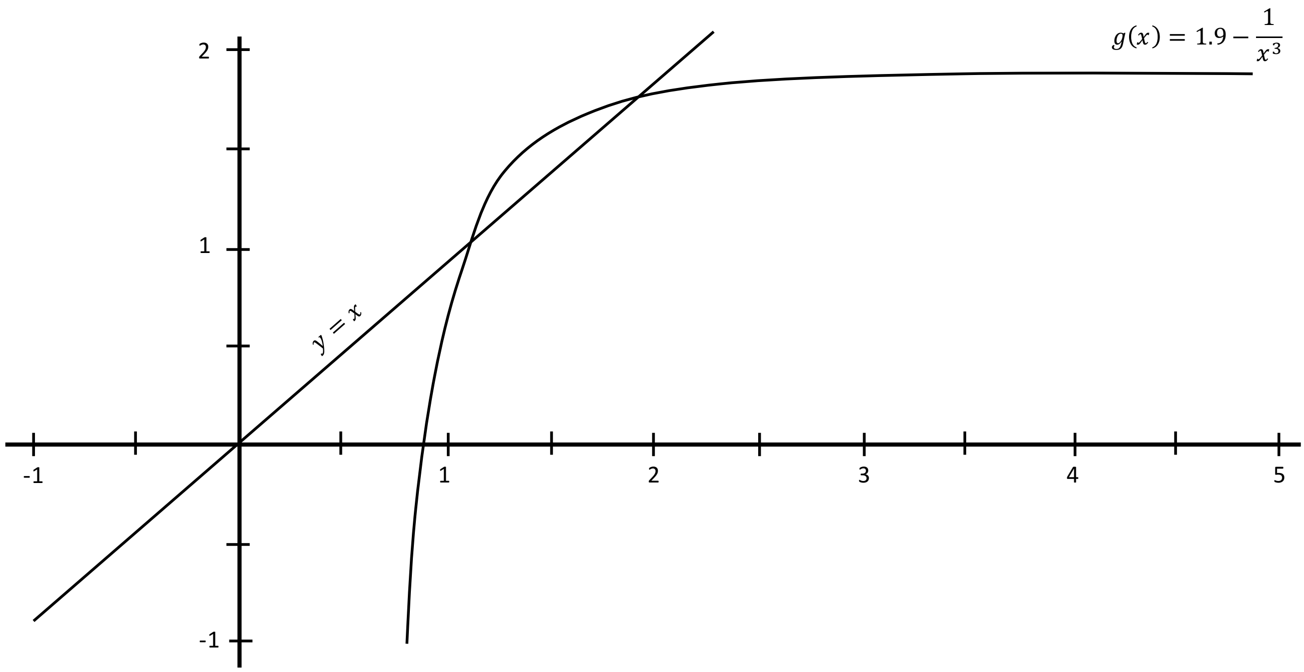
11. Suppose that the following matrix A has eigenvalues at 7 and -1, what are the possible values for k and c:

$$A = \begin{bmatrix} 1 & 3 \\ k & c \end{bmatrix}$$

A. $k = -4, c = 5$
B. $k = 0, c = 2$
C. $k = 4, c = 5$
D. $k = 1, c = -1$
E. $k = -5 + i, c = 4 + i$
F. Cannot determine a value for both k and c because the matrix is a square matrix.
G. $k = 4c$

Question 2: (2 points)

A. The graph below depicts the situation of finding the two roots of $f(x) = 1.9 - \frac{1}{x^3} - x$ using the fixed-point iteration method via the arrangement: $g(x) = 1.9 - \frac{1}{x^3}$. By drawing on the graph neatly, show how each of the following initial points ($x_0 = 0.75, x_0 = 4$) converge or diverge to or from any of the roots.



B. Prove that Newton's Method applied to $f(x) = ax + b$ converges in one step.

Question 3: (4 points)

Consider the root finding problem $f(x) = x^2 - 5 = 0$:

A. Write down the Secant method formula and simplify it as much as possible.

B. Using $x_0 = 2.00$, $x_1 = 2.10$, use the Secant method to find x_2.

Question 4: (3 points)
For the given equation $f(x) = e^x - 2x - 1 = 0$, if $x_0 = 1.5$

a. Give one rearrangement of the equation.
b. Find the value of the root after two iterations using the arrangement you found.
c. What is the actual error **<u>at second iteration</u>** knowing that $f(1.25643) = 0$.