CPE 310C: Numerical Analysis for Engineers

Second Exam, April 15, 2017

- This is a CLOSED BOOK exam. Textbooks, notes, laptops, cell phones, and Internet access are NOT allowed.
- It is a **60-minute** exam, with a total of **20 marks**. There are **20 questions plus 1 bonus question**, and **7 pages** (including this cover page).
- All your answers to multiple choice questions must be marked on this answer sheet. We will **not** take into consideration anything written on the question booklet or if multiple markings are made on the answer sheet. Make sure to mark only one answer.

GOOD LUCK

1	A	B	C	D	E	F	**12**	A	B	C	D
2	A	B	C	D			**13**	A	B	C	D
3	A	B	C	D			**14**	A	B	C	D
4	A	B	C	D			**15**	A	B	C	D
5	A	B	C	D			**16**	A	B	C	D
6	A	B	C	D			**17**	A	B	C	D
7	A	B	C	D			**18**	A	B	C	D
8	A	B	C	D			**19**	A	B	C	D
9	A	B	C	D			**20**	A	B	C	D
10	A	B	C	D			**21**	A	B	C	D
11	A	B	C	D			**22**	A	B	C	D

Total Grade
/20

Name: ______________________________

Student I.D.: ______________________________

Serial No

Divided Difference Interpolating Polynomial

$$P_n(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) + \cdots + a_n(x - x_0)(x - x_1)\cdots(x - x_{n-1})$$

1. Which of the following is your class _______________

   (A) Dr. Ahmed Tamrawi (9:30am - 11:00am)

   (B) Dr. Ola Al-Ta'ani (11:00am - 12:30pm)

   (C) Dr. Hussein Al-Zoubi (8:00am - 9:00am)

   (D) Dr. Hussein Al-Zoubi (9:00am - 10:00am)

   (E) Dr. Ahmed Tamrawi (1:00pm - 2:00pm)

   (F) Dr. Farouq Al-Omari (12:00pm - 1:00pm)

2. For a function f, the divided difference formula gives the interpolation polynomial:
$$P_2(x) = 0.5403 - 0.867(x - 1) - 0.227(x - 1)(x - 1.1)$$
on the data points $x_0 = 1, x_1 = 1.1$, and $x_2 = 1.2$. The value of $\boldsymbol{f(1.2)}$ is:

   (A) 0.36236

   (B) 0.10226

   (C) 0.5403

   (D) 0.227

3. A is a 5×5 matrix and a matrix B is obtained by the row operations of exchanging `Row1` with `Row3`, and then `Row3` is replaced by: $2 \times$ `Row3`. If $det(A) = 17$, then $\boldsymbol{det(B)}$ is:

   (A) 17

   (B) -17

   (C) -34

   (D) 112

4. In cicruit's lab, you had the chance to measure the voltage drop V across a resistor for a number of different values of current i. The results are:

$\boldsymbol{i}$	0.25	0.75	1.25	1.5	2.0
$\boldsymbol{V}$	-0.45	-0.6	0.70	1.88	6.0

   Using **direct interpolation** method to fit a **linear interpolating polynomial**, the voltage drop for $i = 1.3$ is most nearly:

   (A) -0.765

   (B) 0.936

   (C) 0.83

   (D) 0.232

5. The Lagrangian polynomial that passes through the following three data points is given by:

$$P_2(x) = L_0(x)(24) + L_1(x)(37) + L_2(x)(25)$$

$\boldsymbol{x}$	15	18	22
$\boldsymbol{y}$	24	37	25

The value of $L_1(x)$ at $x = 16$ is:

(A) -0.07143

(B) 0.50000

(C) 0.57143

(D) 4.3333

6. The augmented matrix for a 3×3 system of linear equations $A|b = \begin{bmatrix} 2 & 6 & 1 & 8 \\ 0 & 1 & 4 & 9 \\ 4 & 2 & 1 & 4 \end{bmatrix}$. The matrix after eliminating x_2 from the first and the third rows using the Gauss-Jordan elimination **without pivoting** is given by:

(A) $\begin{bmatrix} 1 & 3 & 0.5 & 4 \\ 0 & 1 & 4 & 9 \\ 4 & 2 & 1 & 4 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 & -11.5 & -23 \\ 0 & 1 & 4 & 9 \\ 0 & 0 & 39 & 78 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 0 & -1.75 & -3.5 \\ 0 & 1 & 4 & 9 \\ 0 & 0 & -19.5 & -39 \end{bmatrix}$

7. Assume that $DA = C$ such that $D = \begin{bmatrix} \frac{1}{6} & 0 & 0 \\ 0 & \frac{-1}{2} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 1 & -3 \\ 0.5 & -1 & 2 \\ 0 & 1 & \frac{1}{3} \end{bmatrix}$. What is the value for $\boldsymbol{tr(A)}$? (**Note:** the *trace* of a square matrix is the sum of its diagonal elements)

(A) $\dfrac{-1}{36}$

(B) 0

(C) 15

(D) $\dfrac{8}{6}$

8. Given the two points $(a, f(a))$ and $(b, f(b))$, the **linear Lagrangian polynomial** $P_1(x)$ that passes through these two points is given by:

   (A) $P_1(x) = \dfrac{x-b}{a-b} f(a) + \dfrac{x-a}{a-b} f(b)$

   (B) $P_1(x) = \dfrac{x}{b-a} f(a) + \dfrac{x}{b-a} f(b)$

   (C) $P_1(x) = f(a) + \dfrac{f(b) - f(a)}{b-a}(b-a)$

   (D) $P_1(x) = \dfrac{1}{a-b}[(x-b)f(a) - (x-a)f(b)]$

9. Consider the linear system below, the **maximum error** in evaluating the solution of the system at the second iteration ($x^{(2)}$) using Jacobi method with the initial guess $x^{(0)} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$ and knowing that the solution of the system is $x^{\text{actual}} = [1, 2, -1]^T$:

$$\begin{aligned} 9x_1 + x_2 + x_3 &= 10 \\ 2x_1 + 10x_2 + 3x_3 &= 19 \\ 3x_1 + 4x_2 + 11x_3 &= 0 \end{aligned}$$

   (**Recall** that the *maximum error* is the maximum of actual errors: $(x_i^{\text{actual}} - x_i^{(2)})$)

   (A) $3.22E-1$

   (B) $1.44E-1$

   (C) $5.06E-2$

   (D) $4.02E-2$

10. For which values of $\boldsymbol{\alpha}$ the following system has **no solution**?

$$\begin{aligned} x_1 + 2x_2 - 3x_3 &= 4 \\ 3x_1 - 2x_2 + 5x_3 &= 2 \\ 4x_1 + x_2 + (\alpha^2 - 14)x_3 &= \alpha + 2 \end{aligned}$$

   (A) 1

   (B) 5.2500

   (C) -3.7749

   (D) -2.5414

11. Given that $x^{(1)} = \begin{bmatrix} -2.8333 & -1.4333 & -1.9727 \end{bmatrix}^T$ resulting from applying Gauss-Seidel method using the initial guess $x^{(0)} = \begin{bmatrix} 1 & x_2 & x_3 \end{bmatrix}^T$ and using the following arrangment:

$$x_1 = \frac{1}{6} - \frac{7}{12}x_2 - \frac{1}{4}x_3, \qquad x_2 = -1 - 0.2x_1 - 0.2x_3, \qquad x_3 = -\frac{6}{11} + \frac{2}{11}x_1 + \frac{7}{11}x_2$$

   Then, the initial guess $x^{(0)} = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}$ that was used is most nearly:

   (A) $\begin{bmatrix} 1 & 3 & 5 \end{bmatrix}^T$

   (B) $\begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$

   (C) $\begin{bmatrix} 1 & -0.9046 & -0.8491 \end{bmatrix}^T$

   (D) $\begin{bmatrix} 1 & -1 & -1 \end{bmatrix}^T$

12. Find a relationship between $\boldsymbol{a}$, $\boldsymbol{b}$, and $\boldsymbol{c}$ so that the system is **consistent**:

$$\begin{aligned} x_1 + x_2 + 2x_3 &= a \\ x_1 + x_3 &= b \\ 2x_1 + x_2 + 3x_3 &= c \end{aligned}$$

(A) $a + b + c = 0$

(B) $a - b - c = -1$

(C) $a + b - c = 0$

(D) $a + b - c \neq 0$

13. When the divided differences of order n are all equal to zero, i.e. $f_i^{[n]} = 0$, then the data we used to build the table:

(A) exactly fits a degree n polynomial if $f_i^{[n-1]} > 0$

(B) exactly fits a degree n polynomial if $f_i^{[n-1]} = 0$

(C) exactly fits a degree $n-1$ polynomial if $f_i^{[n-1]} > 0$

(D) exactly fits a degree $n-1$ polynomial if $f_i^{[n-1]} = 0$

14. The actual polynomial f that passes through the following data is given by:

$$f(x) = 8.125x^2 - 324.75x + 3237,\ 15 \le x \le 24$$

$\boldsymbol{x}$	18	22	24
$\boldsymbol{f(x)}$	?	25	123

The corresponding divided difference polynomial is: $b_0 + b_1(x-18) + b_2(x-18)(x-22)$. The value of b_2 is:

(A) 324.75

(B) 8.125

(C) 24

(D) 25

15. If the final matrix after applying Gaussian elimination method is given by $\begin{bmatrix} 25 & 5 & 4 \\ (0.32) & 10.40 & 20.72 \\ (0.4) & (0.5769) & 2.4462 \end{bmatrix}$ Then, the **lower triangual matrix $\boldsymbol{L}$** in the LU decomposition is:

(A) $\begin{bmatrix} 1 & 5 & 4 \\ 0 & 1 & 20.72 \\ 0 & 0 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 25 & 5 & 4 \\ 0 & 10.40 & 20.72 \\ 0 & 0 & 2.4462 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & 0 & 0 \\ 0.3200 & 0 & 0 \\ 0.4000 & 0.5769 & 0 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 0 & 0 \\ 0.3200 & 1 & 0 \\ 0.4000 & 0.5769 & 1 \end{bmatrix}$

16. Which one of the following is the correct form of A, the matrix of coefficients in the system of equations $Ax = b$, using LU decomposition of A:

(A) $\begin{bmatrix} l_{11} & l_{11}u_{12} & l_{11}u_{13} \\ l_{21} & l_{21}u_{12} + l_{22} & l_{21}u_{13} + l_{22}u_{23} \\ l_{31} & l_{31}u_{21} + l_{32} & l_{31}u_{13} + l_{32}u_{23} + l_{33} \end{bmatrix}$

(B) $\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & l_{21}u_{12} + u_{22} & l_{21}u_{13} + u_{23} \\ l_{31}u_{11} & l_{31}u_{12} + l_{32}u_{22} & l_{31}u_{13} + l_{32}u_{23} + u_{33} \end{bmatrix}$

(C) $\begin{bmatrix} l_{11}u_{11} & l_{11}u_{12} & l_{11}u_{13} \\ l_{21}u_{11} & l_{21}u_{12} + l_{22}u_{22} & l_{21}u_{13} \\ l_{31}u_{11} & l_{31}u_{12} + l_{32}u_{22} & l_{31}u_{13} + l_{32}u_{23} \end{bmatrix}$

(D) $\begin{bmatrix} l_{11} & l_{11}u_{12} & l_{11}u_{13} \\ l_{21} & l_{21}u_{12} + l_{22}u_{22} & l_{21}u_{13} + l_{22}u_{23} \\ l_{31} & l_{31}u_{21} + l_{32}u_{22} & l_{31}u_{13} + l_{32}u_{23} + l_{33}u_{33} \end{bmatrix}$

17. Given $n+1$ data pairs, a unique polynomial of degree _______________ passes through $n+1$ data points.

(A) $n+1$

(B) $n+1$ or less

(C) n

(D) n or less

18. Using the divided differences method to find **quadratic polynomial** $P_2(x)$ to fit the following data, the value of $P_2(0.5)$ is:

(A) 0.775

(B) 0.75

(C) 1.35

(D) 0.43

$\boldsymbol{x}$	0.3	0.6	0.9
$\boldsymbol{f(x)}$	0.43	1.12	3.07

19. While solving a system of linear equations by Gauss-Jordon elimination method, if there are still zeros on the main diagonal after all the elementary row operations. Which of the following is **true** about the system?

(A) It has a unique solution

(B) It has no unique solution

(C) The upper triangular matrix after Gauss elimination has no zeroes on the diagonal

(D) We cannot know about the solution without applying Gauss elimination to the system

20. Which of the following statements is **incorrect** about iterative methods of finding roots?

(A) More economical in terms of memory requirements than direct methods

(B) They are self-correcting if an error is made at an iteration

(C) Examples of such methods include: fixed-point iteration, Jacobi, and Gauss-Seidel

(D) They become slower in convergence when the coefficient matrix is sparse (has many zeros)

21. The process of finding the values **outside** the interval (x_0, x_n) is called ______________

(A) interpolation

(B) extrapolation

(C) iterative

(D) polynomial equation

22. (*Bonus Question*) A civil engineer involved in construction requires 4800, 5800, and 5700 m^3 of sand, fine gravel, and coarse gravel, respectively, for a building project. There are three pits from which these materials can be obtained. The composition of these pits is:

	Sand	**Fine Gravel**	**Coarse Gravel**
Pit 1 (p_1)	52%	30%	18%
Pit 2 (p_2)	20%	50%	30%
Pit 3 (p_3)	25%	20%	55%

How many cubic meters must be hauled from each pit in order to meet the engineer's needs? The equations in a matrix form are:

(A) $$\begin{bmatrix} 52 & 20 & 25 \\ 30 & 50 & 20 \\ 18 & 30 & 55 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} = \begin{bmatrix} 4800 \\ 5800 \\ 5700 \end{bmatrix}$$

(B) $$\begin{bmatrix} 0.52 & 0.20 & 0.25 \\ 0.30 & 0.50 & 0.20 \\ 0.18 & 0.30 & 0.55 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} = \begin{bmatrix} 4800 \\ 5800 \\ 5700 \end{bmatrix}$$

(C) $$\begin{bmatrix} 52 & 30 & 18 \\ 20 & 50 & 30 \\ 25 & 20 & 55 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} = \begin{bmatrix} 4800 \\ 5800 \\ 5700 \end{bmatrix}$$

(D) $$\begin{bmatrix} 0.52 & 0.30 & 0.18 \\ 0.20 & 0.50 & 0.30 \\ 0.25 & 0.20 & 0.55 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} = \begin{bmatrix} 4800 \\ 5800 \\ 5700 \end{bmatrix}$$