CPE 310 Quiz 02: Interpolation and Curve Fitting

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Question 1 We are given the following data about a polynomial $P(x)$ of unknown degree constructed using difference interpolation: $P(0) = 2, P(1) = -1, P(2) = 4$. Find the coefficient of x^2 in $P(x)$ if all the third-order differences are 1.

> Note that the difference interpolation or Newton-Gregory interpolation polynomial is given by:
> $P_n(x) = f_i + s\Delta f_i + \frac{s(s-1)}{2!}\Delta^2 f_i + \frac{s(s-1)(s-2)}{3!}\Delta^3 f_i + \cdots + \frac{s(s-1)\cdots(s-n+1)}{n!}\Delta^n f_i$

Question 2 Suppose that the least-squares approximations method is used to fit some tabular data with a polynomial of degree 2, we found the errors as shown in the table below. Find the variance σ^2.

$\boldsymbol{x}$	1	2	3	4	5
$\boldsymbol{e_i}$	0.02	0.02	0.02	0.01	0.03

> Note that $\sigma^2 = \frac{\sum e_i^2}{N-n-1}$

Question 3 Write the normal equations in matrix form for fitting a polynomial of degree 2 using the least-squares approximations method.

$\boldsymbol{x}$	1	2	3	4	5
$\boldsymbol{f(x)}$	1	4	9	16	25

> The normal equations for a polynomial $P(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$ of degree n using least-squares approxmiation is given by:

$$\begin{bmatrix} N & \sum x_i & \sum x_i^2 & \sum x_i^3 & \cdots & \sum x_i^n \\ \sum x_i & \sum x_i^2 & \sum x_i^3 & \sum x_i^4 & \cdots & \sum x_i^{n+1} \\ \sum x_i^2 & \sum x_i^3 & \sum x_i^4 & \sum x_i^5 & \cdots & \sum x_i^{n+2} \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ \sum x_i^n & \sum x_i^{n+1} & \sum x_i^{n+2} & \sum x_i^{n+3} & \cdots & \sum x_i^{2n} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \begin{bmatrix} \sum Y_i \\ \sum x_i Y_i \\ \sum x_i^2 Y_i \\ \vdots \\ \sum x_i^n Y_i \end{bmatrix}$$